

PHENOMENOLOGY OF MOND AND GRAVITATIONAL POLARIZATION*

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The phenomenology of MOND (flat rotation curves of galaxies, baryonic Tully-Fisher relation, *etc.*) is a basic set of phenomena relevant to galaxy dynamics and dark matter distribution at galaxy scales. Still unexplained today, it enjoys a remarkable property, known as the *dielectric analogy*, which could have far-reaching implications. In the present paper we discuss this analogy in the framework of simple non-relativistic models. We show how a specific form of dark matter, made of two different species of particles coupled to different Newtonian gravitational potentials, could permit to interpret in the most natural way the dielectric analogy of MOND by a mechanism of gravitational polarization.

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I. MOTIVATIONS FOR MOND

The Modified Newtonian Dynamics (MOND) was introduced more than 30 years ago by Milgrom [1–3] as an alternative to dark matter, designed to explain a variety of phenomena taking place at the scale of galaxies, which are now collectively referred to as the phenomenology of MOND (see Refs. [4, 5] for reviews). The ability of MOND at reproducing this phenomenology is astonishing, and it is fair to say that this still represents a complete mystery today.

The rotation curves of almost all spiral galaxies are reproduced in great details with a single-parameter fit — the mass-to-luminosity ratio which is *a posteriori* seen to be consistent with the expectations coming from stellar populations. The baryonic Tully-Fisher (BTF) relation [6, 7], an empirical relation between the asymptotic rotation velocity and the baryonic mass of galaxies, and valid for a large range of masses of galaxies [5], is naturally reproduced. In particular, for dwarf galaxies dominated by the gas there is little uncertainty on both the rotation velocity and the baryonic mass, so the evidence for the BTF relation is very strong [8]. The *original sin* of MOND is Milgrom's law, namely that the discrepancy between the dynamical and luminous masses, *i.e.* the presence of dark matter, is correlated with the involved scale of acceleration or magnitude of the gravitational field, see Fig. 1 which is taken from Ref. [5].

In this paper we shall adopt for MOND the modified

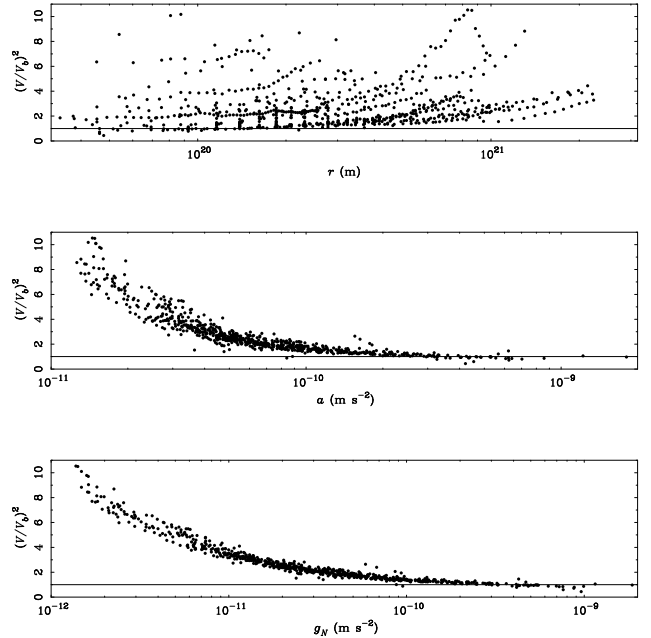


FIG. 1: The mass discrepancy, defined by the ratio $(V/V_b)^2$ where V is the observed velocity and V_b is the velocity attributable to visible baryonic matter, in spiral galaxies. No correlation is found with the distance scale (top panel); however a strong correlation is seen with the acceleration scale (middle panel) and with the gravitational field scale (bottom panel). The mass discrepancy (*i.e.* the presence of dark matter) appears below the critical acceleration scale $a_0 \sim 10^{-10} \text{ m/s}^2$.

Poisson equation¹ of Bekenstein & Milgrom [9]

$$\nabla \cdot \left[\mu \left(\frac{g}{a_0} \right) \mathbf{g} \right] = -4\pi G \rho_b, \quad (1.1)$$

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¹ Boldface letters indicate ordinary Euclidean vectors; G is Newton's gravitational constant.

where ρ_b denotes the ordinary mass density of the baryons. The gravitational field is irrotational, $\mathbf{g} = \nabla U$, with U the gravitational potential, and we denote its norm by $g = |\mathbf{g}|$. The function μ of the ratio g/a_0 is the MOND interpolating function, which interpolates between the Newtonian regime $g \gg a_0$ for which $\mu \simeq 1$ (thus one recovers the usual Poisson equation of Newtonian gravity in this regime), and the MOND weak-field regime $g \ll a_0$ for which μ is linear in its argument, $\mu \simeq g/a_0$. The constant a_0 represents the MOND acceleration scale separating the two regimes as evidenced by Fig. 1.

Several relativistic MOND theories, extending general relativity with new fields and without the need of dark matter, have been proposed:

- The Tensor-Vector-Scalar (TeVeS) theory, which extends general relativity with a time-like vector field and one scalar field [10–12];
- Einstein-æther theories, originally motivated by the phenomenology of Lorentz invariance violation [13, 14], involve a unit time-like vector field non-minimally coupled to the metric and with a non-canonical kinetic term [15, 16];
- A bimetric theory of gravity in which the two metrics are coupled through the difference of their Christoffel symbols [17, 18];
- A variant of TeVeS using a Galileon field and a Vainshtein mechanism to prevent deviations from general relativity at small distances [19];
- A theory based on a preferred time foliation labelled by the so-called Khronon scalar field [20, 21].

The cosmology of these theories has been extensively investigated, notably in TeVeS and non-canonical Einstein-æther theories [22–25]. However, all these theories have difficulties in reproducing the CMB spectrum, even when adding a component of hot dark matter [22].

II. THE DIELECTRIC ANALOGY OF MOND

MOND enjoys a remarkable property, known as the dielectric analogy, which could have far-reaching implications. Indeed the MOND equation (1.1) represents exactly the gravitational analogue (in the non-relativistic approximation) of the Gauss equation of electrostatics when modified by polarization effects taking place in non-linear dielectric media [26]. Taking this analogy at face, we can interpret the MOND function μ entering Eq. (1.1) as a coefficient characterizing some “digravitational medium”, and write it as

$$\mu = 1 + \chi, \quad (2.1)$$

where χ would represent the gravitational susceptibility of this medium, parametrizing the relation between the

polarization, say \mathbf{P} , and the gravitational field,

$$\mathbf{P} = -\frac{\chi}{4\pi G} \mathbf{g}. \quad (2.2)$$

Thus χ characterizes the response of the digravitational medium to an applied gravitational field. From Eq. (1.1) we see that the susceptibility coefficient depends on the norm g of the gravitational field, in close analogy with the electrostatics of non-linear media. The mass density associated with the polarization is given by the same formula as in electrostatics,

$$\rho_{\text{pol}} = -\nabla \cdot \mathbf{P}. \quad (2.3)$$

With those notations Eq. (1.1) can be rewritten as

$$\Delta U = -4\pi G (\rho + \rho_{\text{pol}}), \quad (2.4)$$

indicating that the Newtonian law of gravity may not be violated, but that we are facing a new form of dark matter, in the form of polarized masses with density ρ_{pol} .

Let us proceed further and view the dark matter medium as consisting of individual dipole moments \mathbf{p} with number density n , so that the polarization reads

$$\mathbf{P} = n \mathbf{p}. \quad (2.5)$$

We suppose that the dipoles are made of a doublet of sub-particles, one with positive gravitational mass $m_g = +m$ and one with negative gravitational mass $m_g = -m$, in analogy with electric charges. If the two masses are separated by the spatial vector $\boldsymbol{\xi}$, pointing in the direction of the positive mass, the dipole moment is

$$\mathbf{p} = m \boldsymbol{\xi}. \quad (2.6)$$

Let us further suppose, still with analogy with electric charges, that the sub-particles have positive inertial masses $m_i = m$, so that the dipole moment consists of an ordinary particle $(m_i, m_g) = (m, m)$ associated with an exotic one $(m_i, m_g) = (m, -m)$.

The ordinary particle will always be attracted by some mass distribution made of ordinary matter, while the other particle $(m_i, m_g) = (m, -m)$ will always be repelled by the same mass distribution. In addition the two sub-particles would repel each other. We see therefore that the gravitational dipole is unstable, and we need to invoke a non-gravitational internal force to supersede the gravitational force between the sub-particles [26].

Simply from these considerations we expect that an external gravitational field will exert a torque on the dipole moment in such a way that its orientation will have the positive mass oriented in the direction of the external mass, and the negative one oriented in the opposite direction. Thus we find that \mathbf{p} and \mathbf{P} should point towards the external mass, *i.e.* be oriented in the same direction as the gravitational field \mathbf{g} . From Eq. (2.2) we therefore conclude that

$$\chi < 0. \quad (2.7)$$

This corresponds to an “anti-screening” of ordinary masses by the polarization masses, and an enhancement of the gravitational field in the presence of the digravitational medium. The result (2.7) is nicely compatible with the prediction of MOND, since we have $\mu = 1 + \chi \sim g/a_0 \ll 1$ in the MOND regime. The phenomenology of MOND can thus be interpreted (at the non-relativistic level) as resulting from an effect of gravitational polarization, of some cosmic fluid made of polarizable dipole moments, aligned with the gravitational field of ordinary matter (galaxies), and representing a new form of dark matter.

Of course the previous interpretation of dark matter rings a bell, and it is tempting to interpret this polarizable medium as a sea of virtual pairs of particles and antiparticles. Although this idea poses a lot of problems, let us examine a few orders of magnitude that such an hypothetical medium would have. We thus suppose that the dark matter medium is made of virtual particle-antiparticle pairs $(m, \pm m)$, with polarisation field (2.5) and individual dipole moments (2.6). The classical separation between particles and antiparticles should be of the order of magnitude of the Compton wavelength, hence $\xi \sim \lambda_C \equiv \hbar/mc$, and thus

$$P \sim \frac{n\hbar}{c}. \quad (2.8)$$

On the other hand, in the MOND regime we have $g \simeq a_0$, so from Eq. (2.2) with $\chi \simeq -1$ the polarisation field is of order $P \simeq a_0/4\pi G$, hence we obtain the following estimation of the medium density,

$$n \sim \frac{a_0 c}{4\pi G \hbar} \sim 4.3 \times 10^{35} \text{ cm}^{-3}, \quad (2.9)$$

where we have adopted the common value of the MOND acceleration $a_0 = 1.2 \times 10^{-10} \text{ m/s}^2$. This gives a characteristic length for the separation inside pairs,

$$\xi \sim n^{-1/3} \sim 1.3 \times 10^{-12} \text{ cm}, \quad (2.10)$$

and an estimation of the mass of the dark matter particles,

$$m \sim \frac{\hbar}{\xi c} \sim 14 \text{ MeV}. \quad (2.11)$$

Interestingly, these estimations, in which the value of MOND’s acceleration scale a_0 plays the crucial role, turn out to be very close to typical estimations for the standard QCD vacuum [27, 28]. Regardless of this fact being a coincidence or not, recall that here we made the wild assumption that antiparticles have mass $(m_i, m_g) = (m, -m)$ which is at odds with all theoretical expectations [29]. Furthermore this assumption is severely constrained by equivalence principle Eötvös-type experiments using the virtual e^+e^- and $q\bar{q}$ pairs in ordinary materials [30]. Note also that the above description of vacuum fluctuations based on Compton’s separation is merely semi-classical and probably oversimple.

III. DIPOLAR DARK MATTER AND MODIFIED GRAVITY

Some aspects of the previous model have been promoted to a relativistic description in the concept of dipolar dark matter — a form of matter described by a relativistic current and endowed with a space-like vector field called the dipole moment, and obeying a specific Lagrangian in standard general relativity [31, 32]. But obviously, because of the negative masses, not all aspects of the model could be made compatible with general relativity, in particular it was impossible to give to the dipole moment a microscopic interpretation in terms of sub-particles.

In the present section we shall point out that, in certain conditions, it is possible to mimic the effect of gravitational polarization (and the involved anti-gravity) by coupling the two species of sub-particles to two different Newtonian potentials. We shall provide a non-relativistic model and show how it recovers exactly the MOND equation (1.1) in all non-spherical and dynamical situations. Furthermore this new model will be amenable to a relativistic extension based on a bimetric coupling of dark matter particles [33].

We consider the following non-relativistic Lagrangian for the dynamics of matter fields, consisting of ordinary baryons and two species of dark matter particles, and coupled to gravity:

$$L = \int d^3\mathbf{x} \left\{ -\frac{1}{\kappa} |\nabla U|^2 - \frac{1}{\kappa} |\nabla \underline{U}|^2 - \frac{1}{2\varepsilon} |\nabla(U + \underline{U})|^2 \right. \\ \left. + \rho_b \left(U + \frac{\mathbf{v}_b^2}{2} \right) + \rho \left(U + \phi + \frac{\mathbf{v}^2}{2} \right) \right. \\ \left. + \underline{\rho} \left(\underline{U} - \phi + \frac{\underline{\mathbf{v}}^2}{2} \right) + \frac{a_0^2}{2\alpha} W(X) \right\}. \quad (3.1)$$

Here κ , ε and α denote some coupling constants to be specified later, and a_0 is the MOND acceleration constant scale. The matter fields are described by their usual Newtonian mass density and velocity: (ρ_b, \mathbf{v}_b) for the baryons, (ρ, \mathbf{v}) and $(\underline{\rho}, \underline{\mathbf{v}})$ for respectively the first and second types of dark matter particles. These variables are linked by the ordinary continuity equation, *e.g.* $\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$. In this model, the main point is that the particles (ρ, \mathbf{v}) are coupled to the ordinary Newtonian potential U as for the baryons, but that the particles $(\underline{\rho}, \underline{\mathbf{v}})$ are coupled to a different potential \underline{U} . The two potentials U and \underline{U} interact with each other in the way specified by their kinetic terms in (3.1).

As in the model of Ref. [26], we need to introduce an internal force to stabilize the dipole moment. This is described here by a scalar potential ϕ obeying a non-canonical kinetic term given by the last term in (3.1), which involves a function W of the ratio

$$X \equiv \frac{|\nabla \phi|^2}{a_0^2}. \quad (3.2)$$

This function is determined phenomenologically so as to recover the MOND phenomenology, but in principle it

should be derived from some more fundamental theory. In the limit when $X \rightarrow 0$, which will correspond to the MOND regime, we impose

$$W = X - \frac{2}{3}X^{3/2} + \mathcal{O}(X^2). \quad (3.3)$$

On the other hand, in order to recover the Newtonian limit, it will be sufficient to impose that $W' \equiv dW/dX$ tends to zero when $X \rightarrow +\infty$. We can already note that a stronger condition when $X \rightarrow +\infty$, namely

$$W = A + \frac{B}{X} + \mathcal{O}\left(\frac{1}{X^2}\right), \quad (3.4)$$

where A and B are some constants, will actually be better in order to suppress all polarization effects in the Newtonian regime.

We now vary the Lagrangian with respect to all particles and fields. The equation of motion of baryons is standard,

$$\frac{d\mathbf{v}_b}{dt} = \nabla U. \quad (3.5)$$

At the contrary, because of the postulated internal potential interaction ϕ , we obtain for the dark matter particles,

$$\frac{d\mathbf{v}}{dt} = \nabla(U + \phi), \quad (3.6a)$$

$$\frac{d\underline{\mathbf{v}}}{dt} = \nabla(\underline{U} - \phi). \quad (3.6b)$$

Varying with respect to ϕ we get

$$\nabla \cdot [W'(X) \nabla \phi] = \alpha(\rho - \underline{\rho}). \quad (3.7)$$

Finally, varying with respect to U and \underline{U} we get two equations, which can conveniently be re-arranged into

$$\Delta U = -\frac{\kappa^2}{4(\kappa + \varepsilon)} \left[\left(1 + \frac{2\varepsilon}{\kappa}\right)(\rho_b + \rho) - \underline{\rho} \right], \quad (3.8a)$$

$$\Delta(U + \underline{U}) = -\frac{\kappa\varepsilon}{2(\kappa + \varepsilon)}(\rho_b + \rho + \underline{\rho}). \quad (3.8b)$$

The condition under which our model will work, *i.e.* where a mechanism of gravitational polarization will show up, is

$$\varepsilon \ll \kappa. \quad (3.9)$$

As is already seen at the level of the Lagrangian (3.1), such a condition in the coupling constants forces the two potentials U and \underline{U} to be (approximately) opposite to each other. Therefore, under this condition, we obtain the following Poisson equation for the ordinary Newtonian potential U felt by the baryonic matter,

$$\Delta U = -\frac{\kappa}{4}(\rho_b + \rho - \underline{\rho}), \quad (3.10)$$

the potential in the other sector being given by $\underline{U} = -U$.

We now look for a plasma-like solution of these equations. For this purpose, we assume the existence of an equilibrium configuration with uniform density ρ_0 , and that the two dark matter fluids are displaced with respect to this equilibrium. Their densities can thus be written as

$$\rho = \rho_0 - \frac{1}{2} \nabla \cdot \mathbf{P}, \quad (3.11a)$$

$$\underline{\rho} = \rho_0 + \frac{1}{2} \nabla \cdot \mathbf{P}. \quad (3.11b)$$

Here we defined the polarization $\mathbf{P} = \rho_0 \boldsymbol{\xi}$ where $\boldsymbol{\xi}$ denotes the Eulerian relative displacement; thus, $\rho_0 = nm$ in the notation of Eqs. (2.5)–(2.6). Using (3.11) we can solve for the internal field equation (3.7),

$$\alpha \mathbf{P} = -W' \nabla \phi, \quad (3.12)$$

which shows that the polarization is aligned with the internal field. However, it is not that obvious that it will also be aligned with the gravitational field (“gravitational polarization”). This will come from the equations of motion of the dark matter particles which now read (since $\underline{U} = -U$)

$$\frac{d\mathbf{v}}{dt} = \nabla(U + \phi), \quad (3.13a)$$

$$\frac{d\underline{\mathbf{v}}}{dt} = -\nabla(U + \phi). \quad (3.13b)$$

As we see with this mechanism, the effective ratio between the gravitational mass and the inertial mass of these particles appears to be $m_g/m_i = \pm 1$, in agreement with the picture proposed in Sec. II. However, with this new description the non-relativistic Lagrangian can be generalized to a relativistic formulation [33].

Considering now the relative acceleration combined with Eq. (3.12), we obtain an harmonic oscillator for the polarization \mathbf{P} (or equivalently the displacement $\boldsymbol{\xi}$) embedded in the gravitational field $\mathbf{g} = \nabla U$,

$$\frac{d^2 \mathbf{P}}{dt^2} + \omega_0^2 \mathbf{P} = 2\rho_0 \mathbf{g}. \quad (3.14)$$

The dark matter medium undergoes oscillations with plasma frequency²

$$\omega_0 = \sqrt{\frac{2\alpha \rho_0}{W'}}. \quad (3.15)$$

This imposes the coupling constant α to be positive. Note that from Eq. (3.3) we have $W' = 1 + \mathcal{O}(X^{1/2})$

² Of course this is analogous to the classic derivation of the plasma frequency, see *e.g.* Ref. [34].

which is positive in the MOND regime. Finally, averaging over the plasma oscillations we obtain that the polarization is indeed aligned with the local gravitational field, *i.e.* $\mathbf{P} = 2\rho_0 \mathbf{g}/\omega_0^2$, or equivalently

$$\mathbf{P} = \frac{W'}{\alpha} \mathbf{g}. \quad (3.16)$$

Comparing with (3.12) we see that this simply means that $\nabla\phi = -\mathbf{g}$, which can also be deduced from Eqs. (3.13) when the particles are in average at rest.

Finally the MOND equation follows immediately from the Poisson equation (3.10). By inserting Eqs. (3.11) in Eq. (3.10) we transform it into

$$\nabla \cdot \left[\mathbf{g} - \frac{\kappa}{4} \mathbf{P} \right] = -\frac{\kappa}{4} \rho_b, \quad (3.17)$$

which really looks like an ordinary Poisson equation modified by polarization effects. With the constitutive relation (3.16) we recover the Bekenstein & Milgrom [9] form,

$$\nabla \cdot \left[\mu\left(\frac{g}{a_0}\right) \mathbf{g} \right] = -\frac{\kappa}{4} \rho_b, \quad (3.18)$$

with MOND interpolating function $\mu = 1 - \kappa W'/4\alpha$. It is then easy to see that with the postulated form of the function W in Eq. (3.3) and the following values of the coupling constants:

$$\kappa = 4\alpha = 16\pi G, \quad \varepsilon \ll \kappa, \quad (3.19)$$

we recover exactly the MOND regime when $g \ll a_0$, *i.e.*

$$\mu \equiv 1 - W' = \frac{g}{a_0} + \mathcal{O}\left(\frac{g^2}{a_0^2}\right). \quad (3.20)$$

Thus the phenomenology of MOND appears to be a natural prediction of this model. To recover the ordinary Poisson equation in the Newtonian regime it suffices that W' tends to zero when $X \rightarrow \infty$ (where now $X = g^2/a_0^2$). However there may still be a residual polarization in this limit, see Eq. (3.16). As already mentioned, to suppress it we prefer to impose the stronger condition that XW' tends to zero when $X \rightarrow \infty$, for instance the behaviour given by Eq. (3.4).

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